Cf. Proc. IEEE 18th Conference on Local Computer Networks (LCNs), pp. 140-146, September 1993.

Reliability Analysis for FDDI Dual Homing Networks

Gam D. Nguyen

Naval Research Laboratory, Washington DC 20375

Abstract — *Closed-form reliability functions for the well-known dual homing configuration for the Fiber Distributed Data Interface (FDDI) are derived. The new reliability functions are then compared to that of the dual ring configuration, which again show that the dual homing network is often more reliable than the dual ring network; however, the model analysis also shows that dual homing is recommended only if certain network parameter constraints are met.*

1 Introduction

Fiber Distributed Data Interface (FDDI) is a high-speed, fiber-optic token network consisting of 2 counter-rotating rings [1]. In addition to the fault tolerant level provided by the dual rings, the reliability of an FDDI network is enhanced by the use of station bypass switches or concentrators (CONs) [1, 2, 3]. A station equipped with a bypass switch is switched out of the ring when the station experiences a power failure. A CON facilitates the connection of stations to the ring, and also switches out of the ring any faulty station connected to it. The use of reliable CONs to interconnect stations is the heart of the dual homing configuration.

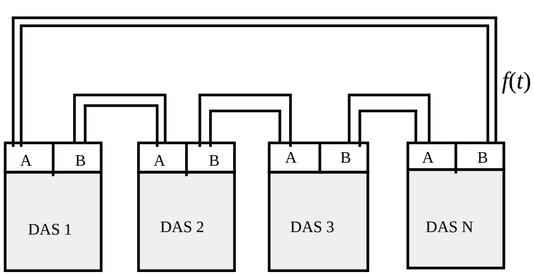

Fig. 1 — Dual ring network

The simplest way to form an FDDI network of N dual attach stations (DASs) is to interconnect the stations as shown in Fig. 1. To use the reliability provided by 2 counter-rotating rings, DASs must be used in the FDDI network. The DASs can be any type of dual attach nodes such as gateways, CONs, or servers. This simple configuration (Fig. 1) is called the *dual ring network*. The reliability function $r(t)$ of the dual ring is given in [2]; suppose that the optical bypass switches in the DASs are working perfectly, then

$$r(t) = N f(t)^{2N-2} - (N-1) f(t)^{2N}, \qquad (1)$$

where N is the number of DASs and $f(t)$ is the reliability function of each of the $2N$ links. A dual homing analogy of (1) is derived in Section 2. Equation (1) is valid under the assumption that the dual ring is reliable as long as the ring is not segmented. Optical bypass switches in DASs switch any station that has no power out of the ring; therefore, the reliability of the ring is not affected unless the number of activated optical bypass switches is excessive. However, the loss caused by activated bypass switches affects both the dual ring and the dual homed network almost equally. This paper will not take the DAS faults into consideration. Keep in mind that our analysis can be extended to incorporate the DAS faults into the model as is done in [2].

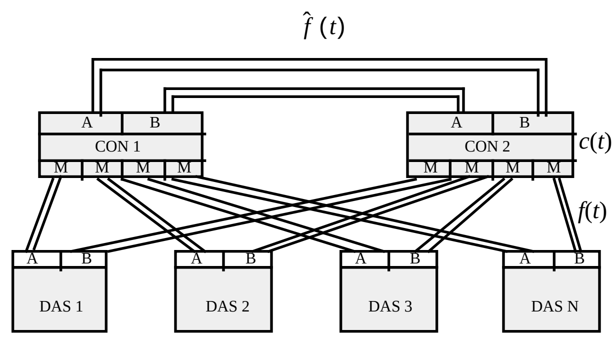

Fig. 2 — Dual homing network

An alternative to the dual ring network is the *dual homing network* (Fig. 2). As expected, the reliability of the dual homing network is improved in many cases by using 2 additional reliable CONs and $4(N + 1)$ fiber links; only $2N$ fiber links are needed in the dual ring network (Fig. 1).

Quantification of this improvement is given in Sections 2 and 3. In addition to providing further reliability, the dual homing network is highly structured and hence facilitates network management as well as network expansion/reduction; stations may be added to or removed from the network without altering its structure.

The FDDI dual homing architecture has been proposed, studied, and implemented for several years. The comparison between dual homing and other types of configurations is reviewed in [3]. The availability of the path between user and network backbone is discussed in [4] for multiple level FDDI dual homing. Our goal is to derive reliability results of the dual homing network in Fig. 2 (and its extensions in Fig. 3). We assume that all components fail independently and that each CON is capable of handling N DASs. Note that the link fault can be any fault that causes the ring to wrap or to be segmented and can include many types such as fiber cuts, severe dB loss, or faulty station transceivers. This simple dual homing model is fundamental for the following reasons:

- The model is the dual homing counterpart of the dual ring shown in Fig. 1. Most FDDI backbone rings can be transformed into either one of these 2 configurations since FDDI standards require that the FDDI backbone be connected by dual attach nodes.
- Many other FDDI configurations involving dual homing can be analyzed by using this dual homing model as the basic building block (Sections 2 and 3); with the use of only one pair of CONs as shown in Fig. 2, the insight can be gained more easily from this simple dual homed network model.
- It is desirable to have a closed-form and simple formula, which is a dual homing counterpart of (1) and can be easily computed (Theorem 1).
- The model analysis shows that dual homing is beneficial only if certain network parameter constraints are met (Theorem 2).

2 Reliability Results for Dual Homing Network of Figure 2

In this section we derive the reliability function for the dual homing network posed in Section 1; the derived formula is illustrated by a numerical example. Then we state a theorem about network parameter constraints for proper dual homing implementations. A more general network model is analyzed in Section 3.

Let $f(t)$, $\hat{f}(t)$, $c(t)$, and $R(t)$ be reliability functions of the link connecting DASs to the CONs, of the link connecting 2 CONs, of the CON, and of the dual homing network respectively (Fig. 2). Furthermore, let N be the number of DASs. The network is said to be operational (i.e., reliable) if there is a communication path among all DASs (i.e., the network is not segmented).

Theorem 1

$$R(t) = \left([2f(t)^2 - f(t)^4]^N [2\hat{f}(t)^2 - \hat{f}(t)^4] + f(t)^{2N} [1 - \hat{f}(t)^2]^2 \right) c(t)^2 + f(t)^{2N} 2c(t) [1 - c(t)] .$$

Proof: Let X be the random variable representing the operational time of the dual homing network shown in Fig. 2. For each $t > 0$, consider 2 mutually exclusive and exhaustive events $A(t)$ and $B(t)$ for the CONs: $A(t)$ is the event that 2 CONs are still operational at time t; and $B(t)$ is the event that only one CON is still operational at time t. Then

$$Pr\{A(t)\} = c(t)^2 \text{ and}$$

$$Pr\{B(t)\} = 2c(t)[1 - c(t)] . \qquad (2)$$

Let $A_1(t)$ be the event that the root ring (i.e., the ring connecting the 2 CONs) is not segmented before t. Then from (1) with $N = 2$

$$Pr\{A_1(t) | A(t)\} = 2\hat{f}(t)^2 - \hat{f}(t)^4 . \qquad (3)$$

Let $A_2(t)$ be the event that the root ring is segmented before t. Then from (3)

$$Pr\{A_2(t) | A(t)\} = 1 - Pr\{A_1(t) | A(t)\} = [1 - \hat{f}(t)^2]^2 . \qquad (4)$$

Then $Pr\{X > t | A(t), A_1(t)\}$ is computed as follows: With the presence of the 2 operational CONs, effectively the fiber links always fail in pairs with a new reliability function $f(t)^2$ for each pair. Each DAS is disconnected from the dual homed network when both pairs (each with reliability function $f(t)^2$) fail; that is, the reliability associated with each DAS is

$$1 - [1 - f(t)^2]^2 = 2f(t)^2 - f(t)^4 .$$

Since there are N such DASs,

$$Pr\{X > t | A(t), A_1(t)\} = [2f(t)^2 - f(t)^4]^N . \qquad (5)$$

Eq. (5) is used several times in the computation in the next section.

For the case where the root ring is segmented, which results in 2 identical rings of single attach stations (SASs), $Pr\{X > t | A(t), A_2(t)\}$ is computed as follows: Note that the second ring is no longer considered reliable because if it is reliable, then all of the links at port B would have to fail simultaneously; this is impossible because *f*(*t*) is associated with a continuous random variable by assumption. Port B links are always active unless there are faults associated with them. Therefore,

$$Pr\{X > t | A(t), A_2(t)\} = f(t)^{2N} \tag{6}$$

From (3) to (6) and the fact that

$$\begin{aligned} Pr\{X > t | A(t)\} \\ &= Pr\{X > t | A(t), A_1(t)\}\, Pr\{A_1(t) | A(t)\} \\ &+ Pr\{X > t | A(t), A_2(t)\}\, Pr\{A_2(t) | A(t)\}\ , \end{aligned}$$

$$Pr\{X > t | A(t)\} =$$

$$\begin{aligned} &[2f(t)^2 - f(t)^4]^N [2f(\hat{t})^2 - f(\hat{t})^4] \\ &+ f(t)^{2N} [1 - \hat{f}(t)^2]^2 . \end{aligned} \tag{7}$$

Note that

$$Pr\{X > t | B(t)\} = f(t)^{2N} . \tag{8}$$

Then from (2), (7), (8) and the fact that

$$\begin{aligned} R(t) = Pr\{X > t\} &= Pr\{X > t | A(t)\}\, Pr\{A(t)\} \\ &+ Pr\{X > t | B(t)\}\, Pr\{B(t)\}\ , \end{aligned}$$

$$\begin{aligned} R(t) = ((2f(t)^2 - f(t)^4)^N [2\hat{f}(t)^2 - \hat{f}(t)^4] \\ + f(t)^{2N} [1 - \hat{f}(t)^2]^2)\, c(t)^2 + f(t)^{2N} 2c(t)\, [1 - c(t)] . \end{aligned}$$

Q.E.D.

Remark 1

- Suppose that 2 perfect CONs with $c(t) = 1$ are used and that the root ring is not segmented (i.e., $\hat{f}(t) = 1$). Then Theorem 1 becomes

$$R(t) = [2f(t)^2 - f(t)^4]^N , \tag{9}$$

  which must be an upper bound for the reliability function $R(t)$ of the dual homing network. This upper bound can be approached as close as desirable by using 2 very reliable CONs and 4 very reliable (e.g., very short) links.

- The reliability function in Theorem 1 depends on $\hat{f}$, f, and c; i.e., $R = R(f, \hat{f}, c)$ (here, for ease of writing, the time variable t is suppressed). Then an upper bound for the reliability function R, which is tighter (and more complicated) than (9), is given by

$$\begin{aligned} R \le min\,(&R(1, \hat{f}, c), R(f, 1, c), R(f, \hat{f}, 1), \\ &R(f, 1, 1), R(1, \hat{f}, 1), R(1, 1, c)) . \end{aligned} \tag{10}$$

  Note that $R(f,1,1)$ is the same as (9).

Example 1

Suppose that $\hat{f}(t) = c(t) = f(t) = e^{-0.001t}$, where t is the time measured, for instance, in days. Thus the mean time to failure (MTTF) of both the CON and fiber link is $1/0.001 = 1000$ days. Then the reliability of the dual ring as well as the dual homing network for various values of t and N (the number of DASs) is given in Table 1 (M is the number of CON pairs, $M = 1$ in this example).

From Table 1, for small network sizes (e.g., $N = 5$), the dual ring network is slightly *more* reliable than the dual homing network since it is less likely that the small dual ring will be segmented. For larger values of N ($N > 9$), dual homing consistently becomes more reliable than dual ring. Finally, as expected, the reliability upper bound value for dual homing is the greatest among the 3 values. The following theorem confirms the superiority of dual homing [see (9)] to dual ring [see (1)] when dual homing is properly implemented [i.e., when $N \ge 4$ and the reliability upper bound (9) is approached].

Theorem 2

(a) $N f^{2N-2} - (N-1) f^{2N} > (2f^2 - f^4)^N$

for $N = 1, 2, 3$ and all $f \quad (0, 1)$. That is, a small dual ring network is more reliable than a small dual homing network with the same number of DASs $N = 1, 2, 3$ [see (9)].

(b) $N f^{2N-2} - (N-1) f^{2N} < (2f^2 - f^4)^N$

for all $N \ge 4$ and all $f \quad [\sqrt{2}/2, 1)$. That is, if $c = 1$ and $\hat{f} = 1$, dual homing is more reliable than dual ring for $N \ge 4$ and $f \ge \sqrt{2}/2$ [see (9)].

Proof: (a) is true for $N = 1, 2$. For $N = 3$, observe that $3(1-f^2)^3 > f^2(1-f^2)^3$, which can be written after some algebra as

$$3f^4 - 2f^6 > (2f^2 - f^4)^3 .$$

Therefore (a) is also true when $N = 3$.

When $N = 4$, (b) becomes $4f^6 - 3f^8 < (2f^2 - f^4)^4$, which can be written after some algebra as

$$0 < (1-f^2)^2[(-4 + 11f^2 - 6f^4) + f^6] ;$$

however, $-4 + 11f^2 - 6f^4 \geq 0$ when $f^2 \geq 1/2$ or $f \geq \sqrt{2}/2 = 0.7071$. Therefore (b) is true at $N = 4$. Note that numerical computation shows that (b) is also true when $N = 4$ if $f = 0.69183$ and is false if $f = 0.69182$. Then (b) is proved for all $N \geq 4$ and all $f \geq \sqrt{2}/2$ by induction as follows. Suppose that (b) is true at N. Then since

$$(1-f^2)^2 = 1 - 2f^2 + f^4 > 0 ,$$

it can be shown after some algebra that

$$[Nf^{2N-2} - (N-1)f^{2N}](2f^2 - f^4) > (N+1)f^{2N} - Nf^{2N+2} ;$$

therefore, from induction hypothesis ((b)),

$$(2f^2 - f^4)^{N+1} > (N+1)f^{2N} - Nf^{2N+2}$$

$$= (N+1)f^{2(N+1)-2} - Nf^{2(N+1)} .$$

Thus (b) is also true at $N+1$; hence the induction proof is completed. Q.E.D.

Theorem 2 shows that dual homing is often more reliable than dual ring; however, to better use the dual homing technique, the following should be met:

- the number of DASs N must be at least 4
- the link reliability $f(t)$ is at least $\sqrt{2}/2 = .7071$
- the ring comprising 2 CONs must be reliable.

Table 1 – Reliability of Dual Homing and Dual Ring Network

$M = 1$	$t = 5$	$t = 10$	$t = 50$
$N = 5$	0.9995[1]	0.9980	0.9555
	0.9990[2]	0.9961	0.9200
	0.9990[3]	0.9962	0.9255
$N = 10$	0.9990	0.9961	0.9130
	0.9980	0.9925	0.8577
	0.9958	0.9841	0.7548
$N = 15$	0.9985	0.9941	0.8724
	0.9971	0.9890	0.8062
	0.9905	0.9653	0.5751
$N = 20$	0.9980	0.9922	0.8336
	0.9962	0.9857	0.7621
	0.9833	0.9411	0.4200
$N = 21$	0.9979	0.9918	0.8261
	0.9960	0.9850	0.7531
	0.9817	0.9358	0.3929
$N = 30$	0.9970	0.9883	0.7612
	0.9945	0.9795	0.6879
	0.9642	0.8814	0.2069
$N = 39$	0.9961	0.9848	0.7013
	0.9929	0.9742	0.6309
	0.9424	0.8196	0.1033
$N = 40$	0.9960	0.9844	0.6950
	0.9928	0.9737	0.6251
	0.9398	0.8124	0.0954
$N = 48$	0.9953	0.9814	0.6462
	0.9915	0.9692	0.5802
	0.9173	0.7542	0.0498
$N = 50$	0.9951	0.9806	0.6345
	0.9912	0.9683	0.5697
	0.9113	0.7395	0.0422

[1] computed from (9): reliability upper bound for dual homing network

[2] computed from Theorem 1: reliability for dual homing network

[3] computed from (1): reliability for dual ring network

3 An Extension

In this section we analyze the dual homing network shown in Fig. 3, which is a natural extension of the network of Fig. 2. Let M be the number of CON pairs; as before, each CON can serve N DASs and has reliability function $c(t)$. The ring consisting of $2M$ CONs is called the *root ring.*

Furthermore, each of the (fiber) links used for interconnecting $2M$ CONs has reliability function $\hat{f}(t)$, and $f(t)$ is the reliability function for each of $4MN$ links used for connecting DASs to CONs. All components are assumed to fail independently. Other definitions from the previous section such as X and $R(t)$ (Theorem 1) are also carried into this section. Note that the number of DASs now becomes MN.

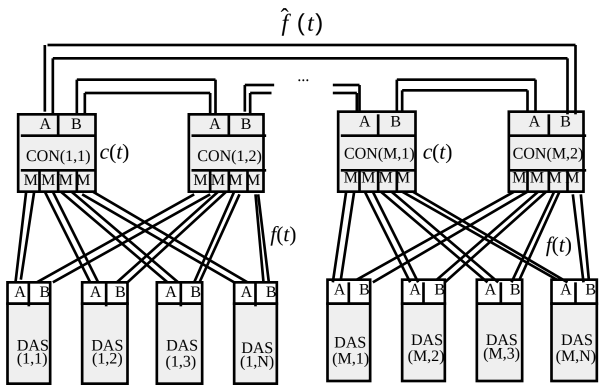

Fig. 3 — Extended dual homing network

Then the reliability function $R(t)$ of the dual homing network shown in Fig. 3 is

$$
\begin{aligned}
Pr\{X > t\} &= Pr\{X > t \mid \text{the root ring is not segmented}\} \\
&\quad \times Pr\{\text{the root ring is not segmented}\} \\
&\quad + Pr\{X > t, \text{the root ring is segmented}\} \quad . \qquad (11)
\end{aligned}
$$

Each of the 2 unknown components in the above summation is computed as follows:

First let L be the number of working CON pairs. Then from the analysis in Section 2 (see (5))

$$
\begin{aligned}
&Pr\{X > t \mid L = M - m, \text{ the root ring is not segmented}\} \\
&\quad = ([2f(t)^2 - f(t)^4]^N)^{M-m} f(t)^{2mN} \text{ and}
\end{aligned}
$$

$$
Pr\{L = M - m\} = \binom{M}{m} c(t)^{2(M-m)} (2c(t)[1 - c(t)])^m .
$$

Therefore from the above 2 equations,

$$
\begin{aligned}
&Pr\{X > t \mid \text{the root ring is not segmented}\} \\
&\quad = \sum_{m=0}^{M} ([2f(t)^2 - f(t)^4]^N)^{M-m} f(t)^{2mN} \\
&\quad \times \binom{M}{m} c(t)^{2(M-m)} (2c(t)[1 - c(t)])^m .
\end{aligned}
$$

Letting $N = 2M$ in (1) gives

$$
\begin{aligned}
&Pr\{\text{the root ring is not segmented}\} \\
&\quad = (1 - 2M)\hat{f}(t)^{4M} + 2M\hat{f}(t)^{4M-2} .
\end{aligned}
$$

From the above 2 equations,

$$
\begin{aligned}
&Pr\{X > t \mid \text{the root ring is not segmented}\} \\
&\quad \times Pr\{\text{the root ring is not segmented}\} \\
&= \sum_{m=0}^{M} ([2f(t)^2 - f(t)^4]^N)^{M-m} f(t)^{2mN} \\
&\quad \times \binom{M}{m} c(t)^{2(M-m)} (2c(t)[1 - c(t)])^m \\
&\quad \times [(1 - 2M)\hat{f}(t)^{4M} + 2M\hat{f}(t)^{4M-2}] . \qquad (12)
\end{aligned}
$$

Then by the same argument used in the previous section, an upper bound for the reliability function $R(t)$ is

$$
R(t) \le [2f(t)^2 - f(t)^4]^{MN} . \qquad (13)
$$

Note that Remark 1 and Theorem 2 remain valid, with slight modification, in this case (see (9)).

The probability $Pr\{X > t, \text{ the root ring is segmented}\}$ is computed as follows, depending on 2 cases: $M = 2$ and $M \ne 2$.

Case 1: $M = 2$. Then there are 4 CONs. Given the fact that $X > t$ and the root ring is segmented, there are only 2 types of ring segmentations:

- The ring is segmented evenly resulting in 2 identical and parallel rings: the first ring consists of CON(1,1) and CON(2,1), and the second ring consists of CON(1,2) and CON(2,2). Note that the second ring is no longer considered reliable because if it is reliable, then all of the links at port B would have to fail simultaneously, which is an impossible event as $f(t)$ is associated with a continuous random variable. Port B links are always active unless there are faults associated with them.
- The ring is segmented unevenly resulting in 2 different rings: one ring has only one CON and the other ring has 3 CONs.

First note that for $L = 0$,

$Pr\{X > t, L = 0,$ the root ring is segmented unevenly$\}$

$= Pr\{X > t | L = 0,$ the root ring is segmented unevenly$\}$

$\times Pr\{$the root ring is segmented unevenly $| L = 0\}$

$\times \; Pr\{L = 0\}$

$$= 4f(t)^{4N} 2c(t)^2 [1 - c(t)]^2 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) .$$

Similarly,

$Pr\{X > t, L = 0,$ the root ring is segmented evenly$\}$

$= Pr\{X > t | L = 0,$ the root ring is segmented evenly$\}$

$\times \; Pr\{L = 0\}$

$\times \; Pr\{$the root ring is segmented evenly $| L = 0\}$

$$= f(t)^{4N} 2c(t)^2 [1 - c(t)]^2 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) . \qquad (14)$$

Summing the above 2 equations gives

$Pr\{X > t, L = 0,$ the root ring is segmented$\}$

$$= 10f(t)^{4N} c(t)^2 [1 - c(t)]^2 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) . \qquad (15)$$

For $L = 1$,

$Pr\{X > t, L = 1,$ the root ring is segmented evenly$\}$

$$= f(t)^{4N} 4c(t)^3 [1 - c(t)] \; (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4)$$

and

$Pr\{X > t, L = 1,$ the root ring is segmented unevenly$\}$

$$= ([2f(t)^2 - f(t)^4]^N f(t)^{2N}) 4c(t)^3 [1 - c(t)] \times [4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4] .$$

Then summing the above 2 equations gives

$Pr\{X > t, L = 1,$ the root ring is segmented$\}$

$$= ([2f(t)^2 - f(t)^4]^N f(t)^{2N} + f(t)^{4N}) \times 4c(t)^3 [1 - c(t)] \; (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) . \qquad (16)$$

For $L = 2$,

$Pr\{X > t, L = 2,$ the root ring is segmented evenly$\}$

$$= f(t)^{4N} c(t)^4 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4)$$

and

$P\{X > t, L = 2,$ the root ring is segmented unevenly$\}$

$$= 4[2f(t)^2 - f(t)^4]^N f(t)^{2N} c(t)^4 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) .$$

Hence by summing the above 2 equations,

$Pr\{X > t, L = 2,$ the root ring is segmented$\}$

$$= (4[2f(t)^2 - f(t)^4]^N f(t)^{2N} + f(t)^{4N}) \times c(t)^4 (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) . \qquad (17)$$

Then by summing (15), (16) and (17):

$Pr\{X > t,$ the root ring is segmented$\}$

$$= \sum_{i=0}^{2} Pr\{X > t, L = i, \text{the root ring is segmented}\}$$

$$= (4(1 - \hat{f}(t))^2 \hat{f}(t)^6 + 4(1 - \hat{f}(t))^3 \hat{f}(t)^5 + \hat{f}(t)^4 (1 - \hat{f}(t))^4) \times \{10f(t)^{4N} c(t)^2 [1 - c(t)]^2 + ([2f(t)^2 - f(t)^4]^N f(t)^{2N} + f(t)^{4N}) 4c(t)^3 [1 - c(t)] + (4[2f(t)^2 - f(t)^4]^N f(t)^{2N} + f(t)^{4N}) c(t)^4\} . \qquad (18)$$

Case 2: $M \neq 2$. Note that the necessary condition for the segmented network to be reliable in this case is that: At most one CON is isolated. Then it can be shown that

$$Pr\{X > t, \text{the root ring is segmented}\}$$

$$= Pr\{X > t, \text{only one CON is isolated}\}$$

$$= \sum_{m=0}^{M} \left([2f(t)^2 - f(t)^4]^N \right)^{M-m-1} f(t)^{2Nm}$$

$$\times \binom{M}{m} c(t)^{2(M-m)} [2c(t)(1-c(t))]^m$$

$$\times \left(4(1-\hat{f}(t))^2 \hat{f}(t)^{4M-2} + 4(1-\hat{f}(t))^3 \hat{f}(t)^{4M-3}\right.$$

$$\left. + (1-\hat{f}(t))^4 \hat{f}(t)^{4M-4}\right)$$

$$\times [f(t)^{2N}(M-m) + m[2f(t)^2 - f(t)^4]^N]. \qquad (19)$$

Combining (11), (12), (18), and (19) yields Theorem 3, which is an extension of Theorem 1.

Theorem 3

$$R(t) = \sum_{m=0}^{M} \left([2f(t)^2 - f(t)^4]^N \right)^{M-m} f(t)^{2mN}$$

$$\times \binom{M}{m} c(t)^{2(M-m)} (2c(t)[1-c(t)])^m$$

$$\times \left((1-2M)\hat{f}(t)^{4M} + 2M\hat{f}(t)^{4M-2}\right) + S(M,t),$$

where $S(M,t) = Pr\{X > t, \text{the root ring is segmented}\}$, which is given by (18) if $M = 2$, and by (19) if $M \neq 2$.

Note that Theorem 3 reduces to Theorem 1 when $M = 1$. Theorem 2 remains valid if N DASs, which were used in that theorem, are replaced by MN DASs in this section.

Example 2

Suppose now that Example 1 is extended to include 6 CONs (i.e., $M = 3$). Then the reliability values computed at $t = 5$, 10, and 50 days are given in Table 2.

Note that, by comparing Table 2 with Table 1, the dual homing network with $M = 1$ is more reliable than that with $M = 3$ even though their reliability upper bounds (9) and (13) are equal.

Table 2 – Reliability of Dual Ring and Extended Dual Homing Network

$M = 3$	$t = 5$	$t = 10$	$t = 50$
$MN = 21$	0.9979[1]	0.9918	0.8261
($N = 7$)	0.9947[2]	0.9795	0.6492
$MN = 39$	0.9961	0.9848	0.7013
($N = 13$)	0.9913	0.9669	0.5137
$MN = 48$	0.9953	0.9814	0.6462
($N = 16$)	0.9897	0.9609	0.4624

[1] computed from (13): upper bound for dual homing network

[2] computed from Theorem 3: extended dual homing network

4 Conclusions

Closed form reliability functions for the FDDI dual homing networks are derived (Theorems 1 and 3). Dual homing is often more reliable than dual ring; however, dual homing technique is beneficial only if the following conditions are met (Theorem 2):

- the number of DASs must be at least 4
- the link reliability should be at least $\sqrt{2}/2 = .7071$
- the root ring comprising CONs must be reliable
- the number of CON pairs on the root ring should be as small as possible.

References

1. F. Ross, “FDDI - A Tutorial,” *IEEE Communications*, **24**(5), pp. 10-17, 1986.

2. D. Logothetis and K. Trivedi, “Reliability Analysis of the FDDI Token Ring,” Proc. 16th IEEE Conference on Local Computer Networks (LCNs), pp. 41-51, October 1991.

3. K. Ochettree, “Using Redundancy in FDDI Networks,” Proc. 15th IEEE Conference on Local Computer Networks (LCNs), pp. 261-267, October 1990.

4. M. Willebeek-LeMair and A. Ruiz, “FDDI Network Availability Analysis,” Proc. 16th IEEE Conference on Local Computer Networks (LCNs), pp. 24-33, October 1991.