TRANSMISSION STRATEGIES FOR SINGLE-DESTINATION WIRELESS NETWORKS

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Abstract We consider the media-access control problem for nodes with heavy traffic in single-destination wireless networks. We assume that each source transmits in each time slot according to a transmission probability, which is a continuous value between 0 and 1. Our goal is to determine the values of the transmission probabilities so that the network throughput is maximized. In this paper, we show that the maximum throughput is achieved only if these values are either 0 or 1. We obtain closed-form results for optimal throughput for networks that operate under a homogenous situation in which the expected value of the received power at the destination is the same for each source. We then extend our studies to more general networks, which rely on exhaustive search for the optimal set of transmissions. The search has exponential complexity and is feasible only for networks with small or moderate sizes. Thus, we also develop heuristic algorithms, which have polynomial-time complexity and are suitable for large and general networks.

1 Introduction

We study the media-access control (MAC) problem for one of the most basic forms of wireless networking: a network with K sources and a single destination. An example network is shown in Fig. 1. Such a single-destination may be part of a much larger multiple-source multiple-destination network (e.g., a cellular network or a sensor network). The network operates in a complex environment that includes receiver noise, other-user interference, and channel fading. Assume that the receiver has multi-packet reception (MPR) capability, i.e., one or more packets may be decoded successfully if the signal-to-interference-plus-noise ratio (SINR) exceeds a specified threshold [3, 4].

We focus on the MAC problem for the uplink channel (i.e., the channel from the sources to the destination), and we are interested in how each of the K sources transmits in each time slot. Here, we study a model that is different from existing models such as random access (e.g., Aloha), CSMA/CD, CSMA/CA (e.g., 802.11) and CDMA [2, 6, 7, 8]. In particular, our model assumes that source i transmits in each time slot according to a transmission probability p_i, which is a continuous value between 0 and 1. Our goal is to determine the vector $(p_1, p_2, \ldots, p_K)$ so that the network throughput is maximized.

In this paper, we show that the maximum throughput is achieved only if the transmission probabilities are either 0 or 1. This result, which holds for any form of fading and receiver noise, implies that the search space for the optimal probabilities is reduced from an infinite and continuous space to a finite set of points. Thus, our problem is equivalent to that of determining a subset of transmission nodes that maximizes the throughput.

We obtain the closed-form optimal solution for networks that operate under a homogenous situation in which the expected value of the received power at the destination is the same for each source. This case may arise when the sources are located at the same distance from the destination, or alternatively when power control is used. We then extend our studies to more general networks, which rely on exhaustive search for the optimal set of transmission nodes. The search has exponential complexity and is feasible only for networks with small or moderate sizes. Thus, we also develop heuristic algorithms, which have polynomial-time complexity and are suitable for large and general networks. Additionally, our numerical results show that these heuristics perform almost identically to the optimal solutions obtained by expensive exhaustive search.

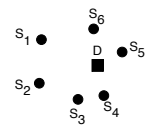

Fig. 1 A network with K sources (S_i) and a single destination (D): Source S_i transmits with probability p_i in a time slot.

2 Network Model and Assumptions

We consider a stationary wireless network that has K sources, denoted by $S_1, S_2, \ldots, S_K$, that transmit their traffic to a common destination, denoted by D. An example network with $K = 6$ sources is shown in Fig. 1. We assume the following:

- The nodes, whose locations are known and fixed, are equipped with omnidirectional antennas.
- The destination can successfully receive more than one transmission at a time, i.e., it has MPR capability.
- Each source can communicate directly with the destination. Routing is not considered in this paper.
- The traffic is heavy in the sense that each source always has traffic to transmit, i.e., its transmission queue is never empty.
- Time is divided into slots. The traffic is expressed in terms of fixed-size packets such that it takes one time slot to transmit one packet.
- Our primary performance measure is sum throughput, which is the average number of packets that are successfully received by the destination per time slot. We do not address issues such as time delays and stability analysis in this paper.
- Source S_i transmits with probability p_i in each time slot, $0 \leq p_i \leq 1$. Our goal is to determine the vector $(p_1, p_2, \ldots, p_K)$ that maximizes the throughput.

We assume that a packet is successfully received, even in the presence of interference and noise, as long as its SINR exceeds a given threshold [3, 4]. More precisely, suppose that we are given a set H of sources that transmit in the same time slot, and $S \in H$. Let $P_{\text{rx}}(S, D)$ be the signal power received by node D for node S, and let $\text{SINR}(S, D)$ be the SINR at D for the transmission from S, i.e.,

$$\text{SINR}(S, D) = \frac{P_{\text{rx}}(S, D)}{P_{\text{noise}}(D) + \sum_{U \in H \setminus \{S\}} P_{\text{rx}}(U, D)}$$

where $P_{\text{noise}}(D)$ denotes the receiver noise power at D. We assume that a packet transmitted by S is successfully received by D if

$$\text{SINR}(S, D) > \beta \tag{1}$$

where $\beta \geq 0$ is a threshold at D, which is determined by application requirements and the properties of the network. When $\beta < 1$ (e.g., in spread-spectrum networks), it is possible for two or more transmissions to satisfy (1) simultaneously.

The wireless channel is affected by fading, as described below. Let $P_{\text{tx}}(S)$ be the transmit power at node S, and $r(S, D)$ be the distance between nodes S and D. When node S transmits, the power received by node D is modeled by

$$P_{\text{rx}}(S, D) = A(S, D) g(S, D)$$

where $A(S, D)$ is a random variable that incorporates the channel fading. We refer to $g(S, D)$ as the "received power factor," which depends on $r(S, D)$ and $P_{\text{tx}}(S)$. For far-field communication (i.e., when $r(S, D) \gg 1$), we have

$$g(S, D) = P_{\text{tx}}(S) r(S, D)^{-a} \tag{2}$$

where a is the path-loss exponent whose typical values are between 2 and 4. A simple approximate model for both near-field (i.e., when $r(S, D) < 1$) and far-field communication is

$$g(S, D) = P_{\text{tx}}(S)[r(S, D) + 1]^{-a} \tag{3}$$

where the expression $r(S, D) + 1$ is used to ensure that $g(S, D) \leq P_{\text{tx}}(S)$. Under Rayleigh fading, $A(S, D)$ is exponentially distributed [9, p. 36].

Our goal is to study methods for accomplishing the communication between the sources and the destination, and to evaluate the resulting performance. Existing methods such as TDMA, 802.11, and Aloha do not allow simultaneous transmissions, i.e., there is exactly one transmission and no other-user interference in each time slot. In this paper we consider MPR approaches, as described in the following sections, under which more than one transmission is allowed in a time slot.

3 Throughput Evaluation

Recall that source S_i transmits with probability p_i in each time slot, $0 \leq p_i \leq 1$. Our goal is to determine the vector $(p_1, p_2, \ldots, p_K)$ that maximizes the network throughput, which is the average number of packets that are successfully received by the destination per time slot. The throughput is computed as follows. To simplify the notation, we define $q_i = 1 - p_i$.

Let H be a set of sources, and let $C_H(S_i, D)$ be the probability that a packet from source S_i is successfully received by destination D in a time slot, given that all the sources in H simultaneously transmit in the slot.

First, consider the simplest case of $K = 2$ sources. The throughput is $T(p_1, p_2) = T_1 + T_2$, where T_i is the throughput contributed by S_i, $i = 1, 2$. The term T_1 is the sum of 2 components: The 1st component is the probability that a packet from source S_1 is successfully received by destination D in a time slot, given that both S_1 and S_2 transmit in the time slot (which occurs with probability $p_1 p_2$). Thus, the 1st component is $C_{\{S_1, S_2\}}(S_1, D) p_1 p_2$. The 2nd component is the probability that a packet from S_1 is successfully received by D in a time slot, given that only S_1 transmits in the time slot (which occurs with probability $p_1 q_2$). Thus, the 2nd component is $C_{\{S_1\}}(S_1, D) p_1 q_1$. We then have $T_1 = C_{\{S_1, S_2\}}(S_1, D) p_1 p_2 + C_{\{S_1\}}(S_1, D) p_1 q_2$. Similarly, it can be shown that $T_2 = C_{\{S_1, S_2\}}(S_2, D) p_1 p_2 + C_{\{S_2\}}(S_2, D) q_1 p_2$. To summarize, the throughput for the case of $K = 2$ is

$$\begin{aligned} T(p_1, p_2) = {} & C_{\{S_1, S_2\}}(S_1, D) p_1 p_2 + C_{\{S_1\}}(S_1, D) p_1 q_2 \\ & + C_{\{S_1, S_2\}}(S_2, D) p_1 p_2 + C_{\{S_2\}}(S_2, D) q_1 p_2 \end{aligned} \tag{4}$$

It can be verified that the right hand side of (4) equals $E(L) = \Pr\{L = 1\} + 2\Pr\{L = 2\}$, where $L \in \{0, 1, 2\}$ is the random variable representing the number of successful transmissions in a time slot. Thus, the throughput $T(p_1, p_2)$ computed in (4) is indeed the average number of successful transmissions in a time slot.

For the case of $K = 3$ sources, it can be shown that the throughput is given by

$$\begin{aligned} & T(p_1, p_2, p_3) = \\ & C_{\{S_1\}}(S_1, D) p_1 q_2 q_3 + C_{\{S_2\}}(S_2, D) q_1 p_2 q_3 + C_{\{S_3\}}(S_3, D) q_1 q_2 p_3 \\ & + C_{\{S_1, S_2\}}(S_1, D) p_1 p_2 q_3 + C_{\{S_1, S_2\}}(S_2, D) p_1 p_2 q_3 \\ & + C_{\{S_1, S_3\}}(S_1, D) p_1 q_2 p_3 + C_{\{S_1, S_3\}}(S_3, D) p_1 q_2 p_3 \\ & + C_{\{S_2, S_3\}}(S_2, D) q_1 p_2 p_3 + C_{\{S_2, S_3\}}(S_3, D) q_1 p_2 p_3 \\ & + C_{\{S_1, S_2, S_3\}}(S_1, D) p_1 p_2 p_3 + C_{\{S_1, S_2, S_3\}}(S_2, D) p_1 p_2 p_3 \\ & \qquad + C_{\{S_1, S_2, S_3\}}(S_3, D) p_1 p_2 p_3 \end{aligned}$$

For the general case of K sources, it can be shown that the throughput is given by

$$T(p_1, p_2, \ldots, p_K) = \sum_{H \neq \emptyset} \prod_{S_i \in H} p_i \prod_{S_j \notin H} (1 - p_j) \sum_{S \in H} C_H(S, D) \tag{5}$$

with the convention $\prod_{S_j \in \emptyset} (1 - p_j) = 1$.

Direct computation of the throughput $T(p_1, p_2, \ldots, p_K)$ as given in (5) requires the summation of terms whose number increases rapidly as K increases. As seen above, the number of terms grows from 4 for $K = 2$ to 12 for $K = 3$. It can be shown in general that the number of terms is $K 2^{K-1}$. In the following, we show that at the point of maximized throughput, p_i must be 0 or 1 for all i, i.e., the computation of throughput can then be greatly simplified.

4 Reduced Search Space for Optimal Solution

The following theorem is general, because it holds for arbitrary values of $C_H(S_i, D)$ used in the evaluation of the throughput $T(p_1, p_2, \ldots, p_K)$ as given in (5), i.e., the theorem does not depend on the form of receiver noise, power attenuation, and fading.

Theorem 1 Assume that each source S_i transmits with probability p_i in a time slot, $0 \le p_i \le 1$. The throughput $T(p_1, p_2, \ldots, p_K)$ is then maximized for some binary vector $(p_1, p_2, \ldots, p_K)$, i.e., p_i must be 0 or 1, for all i. Thus, the optimal solution is confined to a finite set of 2^K elements.

Proof Consider an arbitrary source S_i, which transmits with probability p_i in a time slot, $0 \le p_i \le 1$. For each $(K-1)$-tuple $(p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_K)$, let us study the dependence of the throughput $T_i(p_i) = T(p_1, p_2, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_K)$ on p_i. First, note that $T_i(p_i)$ is a weighted sum of terms, each of which contains either p_i or $(1 - p_i)$. Thus, for each $(K-1)$-tuple $(p_1, p_2, \ldots, p_{i-1}, p_{i+1}, \ldots, p_K)$, $T_i(p_i)$ varies according to a straight line.

Because $0 \le p_i \le 1$, we see that this straight line starts at $p_i = 0$ and ends at $p_i = 1$. Note that the slope of this straight line depends on the values of p_j and $C_H(S_k, D)$ for all $j \ne i$ and all sources S_k. Because $T_i(p_i)$ varies according to a straight line, its maximum must occur at one of its endpoints, i.e., $T_i(p_i) \le \max\{T_i(0), T_i(1)\}$. Thus, we have

$$\begin{aligned} &T(p_1, p_2, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_K) \\ &\quad \le \max\{T(p_1, p_2, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, p_{i-1}, 1, p_{i+1}, \ldots, p_K)\} \end{aligned} \tag{6}$$

Reasoning as above, the terms on the RHS of (6) can be further expanded as

$$\begin{aligned} &T(p_1, p_2, \ldots, p_{i-1}, 0, p_{i+1}, \ldots, p_K) \\ &\quad \le \max\{T(p_1, p_2, \ldots, 0, 0, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, 1, 0, p_{i+1}, \ldots, p_K)\} \end{aligned} \tag{7}$$

and

$$\begin{aligned} &T(p_1, p_2, \ldots, p_{i-1}, 1, p_{i+1}, \ldots, p_K) \\ &\quad \le \max\{T(p_1, p_2, \ldots, 0, 1, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, 1, 1, p_{i+1}, \ldots, p_K)\} \end{aligned} \tag{8}$$

From (6), (7), and (8), we then have

$$\begin{aligned} &T(p_1, p_2, \ldots, p_{i-1}, p_i, p_{i+1}, \ldots, p_K) \\ &\quad \le \max\{T(p_1, p_2, \ldots, 0, 0, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, 1, 0, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, 0, 1, p_{i+1}, \ldots, p_K), \\ &\qquad T(p_1, p_2, \ldots, 1, 1, p_{i+1}, \ldots, p_K)\} \end{aligned} \tag{9}$$

By expanding the terms on the RHS of (9) repeatedly, it can be shown that

$$T(p_1, p_2, \ldots, p_K) \le \max\{T(0, 0, \ldots, 0), \ldots, T(1, 1, \ldots, 1)\}$$

i.e., $T(p_1, p_2, \ldots, p_K)$ is maximized at one of the binary vectors $(0, 0, \ldots, 0), \ldots, (1, 1, \ldots, 1)$.

For the case of $K = 2$, the proof is especially simple, because by applying (8) repeatedly, we have

$$\begin{aligned} T(p_1, p_2) &\le \max\{T(0, p_2), T(1, p_2)\} \\ &\le \max\{\max\{T(0, 0), T(0, 1)\}, \max\{T(1, 0), T(1, 1)\}\} \\ &= \max\{T(0, 0), T(0, 1), T(1, 0), T(1, 1)\} \qquad \square \end{aligned}$$

Note that $p_i = 0$ means source S_i never transmits, and $p_i = 1$ means source S_i always transmits. Theorem 1 states that, when the throughput is maximized, p_i must be either 0 or 1 for all sources S_i, i.e., only a subset of sources are allowed to transmit and all others are blocked from transmissions. Thus, the maximized throughput can be simplified to

$$T = \sum_{S \in H} C_H(S, D) \tag{10}$$

where H is a set of sources that transmit in the time slot (i.e., their transmission probabilities are 1). As seen later, Theorem 1 can be adapted to allow all the sources to transmit in multiple time slots (see Remarks 1 and 2).

Theorem 1 is valid for any form of receiver noise and channel fading. For the rest of this paper, we focus on the case of Rayleigh fading, for which the following result (whose proof is given in [1, 5]) provides the exact formula for $C_H(S, D)$, which depends on the receiver noise, channel fading, receiver threshold, and other-user interference.

Theorem 2 Suppose that the fading between a transmitting node S and a receiving node D is modeled as a Rayleigh random variable Y_S with parameter $v(S, D)$. For $S \ne U$, assume that Y_S and Y_U are independent. Given that all the nodes in H simultaneously transmit in a time slot, the probability that a packet from S is successfully received by D is

$$C_H(S, D) = \frac{\exp\left(-\dfrac{\beta P_{\text{noise}}(D)}{v(S, D) g(S, D)}\right)}{\displaystyle\prod_{U \in H \setminus \{S\}} \left[1 + \beta \frac{v(U, D) g(U, D)}{v(S, D) g(S, D)}\right]}$$

where β and $P_{\text{noise}}(D)$ are the required SINR threshold and the receiver noise power at D, respectively, and $g(S, D)$ is the received power factor defined in (2) and (3).

5 Networking in Homogenous Situation

We now assume that the network operates in a homogenous situation, meaning that packets from all sources have equal chance of being received successfully, i.e., $C_H(S_i, D) = C_H(S_1, D)$ for all subsets of sources H and all i. From (10), the optimal throughput is simply $T = |H| C_H(S_1, D)$ for some subset of sources H.

For the case of Rayleigh fading, it is sufficient to assume that $v(S_i, D) g(S_i, D) = v(S_1, D) g(S_1, D)$ for all i, where $v(S_i, D)$ is the Rayleigh fading parameter and $g(S, D)$ is the received power factor, as defined in (2) and (3), which depends on the distance $r(S, D)$ and the transmitted power $P_{\text{tx}}(S)$. This case may arise when all the sources transmit with the same power and are located the same distance from the destination (as shown in Fig. 2), or alternatively when power control is used. Thus, the expected value of the received power at the destination is the same for each source.

Theorem 3 Assume that $v(S_i, D)g(S_i, D) = v(S_1, D)g(S_1, D)$ for $1 \le i \le K$, and also assume that m sources simultaneously transmit in a time slot, $1 \le m \le K$. The throughput is then given by

$$T = m\frac{\exp\left(-\dfrac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right)}{(1+\beta)^{m-1}}$$

which is maximized when

$$m = \begin{cases} 1 & \text{if } \beta \ge 1 \\ \left\lfloor \dfrac{1}{\ln(1+\beta)} + 0.5 \right\rfloor & \text{if } e^{1/K} - 1 < \beta < 1 \\ K & \text{if } 0 \le \beta \le e^{1/K} - 1 \end{cases}$$

Proof Let H be a set of sources that transmit in a time slot, and let $m = |H|$. Theorem 2 implies that

$$C_H(S_i, D) = C_H(S_1, D) = \frac{\exp\left(-\frac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right)}{(1+\beta)^{m-1}}$$

The total throughput from the m transmissions is then

$$T(m) = m\frac{\exp\left(-\frac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right)}{(1+\beta)^{m-1}}$$

By considering m as a continuous variable, taking the derivative of $T(m)$, and then setting the results to zero, it can be shown that $m = 1/\ln(1+\beta)$.

Note that m in our original problem is an integer between 1 and K. Thus, $m = 1$ if $1/\ln(1+\beta) \le 1$, and $m = K$ if $1/\ln(1+\beta) \ge K$. Further, $m \approx 1/\ln(1+\beta)$ when $1 < 1/\ln(1+\beta) < K$. In this case, our numerical studies suggest that $m = \lfloor 1/\ln(1+\beta) + 0.5 \rfloor$. To summarize, the optimal m is given by

$$m = \begin{cases} 1 & \text{if } 1/\ln(1+\beta) \le 1 \\ \lfloor 1/\ln(1+\beta) + 0.5 \rfloor & \text{if } 1 < 1/\ln(1+\beta) < K \\ K & \text{if } 1/\ln(1+\beta) \ge K \end{cases}$$

which is equivalent to

$$m = \begin{cases} 1 & \text{if } \beta \ge e - 1 \\ \left\lfloor \dfrac{1}{\ln(1+\beta)} + 0.5 \right\rfloor & \text{if } e^{1/K} - 1 < \beta < e - 1 \\ K & \text{if } 0 \le \beta \le e^{1/K} - 1 \end{cases}$$

Now assume that $\beta \ge 1$. It can be shown by induction that $m \le 2^{m-1}$ for $m \ge 1$. Using this inequality, we then have

$$\begin{aligned} T(1) &= \exp\left(-\frac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right) \\ &\ge m\frac{\exp\left(-\frac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right)}{2^{m-1}} \\ &\ge m\frac{\exp\left(-\frac{\beta P_{\text{noise}}}{v(S_1, D)g(S_1, D)}\right)}{(1+\beta)^{m-1}} \\ &= T(m) \end{aligned}$$

i.e., the optimum is $m = 1$ for $\beta \ge 1$. □

Remark 1
(1) Theorem 3 implies that the optimal number of transmissions, m, is independent of P_{noise} for all $\beta \ge 0$. Further, for $\beta \ge 1$, the optimum is $m = 1$, independent of the network size K.
(2) Let us compare the throughput performance of our method with the well-known slotted Aloha for the case of $P_{\text{noise}} = 0$. The throughput of the slotted Aloha is $T_{\text{Aloha}} = 1/e = 0.368$ for sufficiently large K [7]. Theorem 3 implies that the throughput T of our method is between 1 and K. Note that T_{Aloha} is independent of β, while T is highly dependent on β. Further, the slotted Aloha is distributed, while our method is centralized.
(3) Theorem 3 specifies the optimal number of sources, m, that transmit in each time slot. For the case of $m < K$, we can use a round-robin scheme to allow all the sources to transmit in multiple time slots (i.e., a method for fairness). For example, suppose that the total number of sources is $K = 5$, and the optimal number of sources that transmit in each time slot is $m = 3$. All the sources can transmit in multiple time slots as follows: S_1, S_2, S_3 (in slot 1), S_4, S_5, S_1 (in slot 2), S_2, S_3, S_4 (in slot 3), S_5, S_1, S_2 (in slot 4), S_3, S_4, S_5 (in slot 5), etc. □

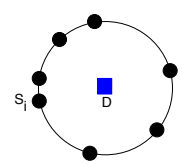

Fig. 2 The sources lie on a circle centered at the destination.

6 Networking in Non-homogenous Situation

We now assume that the network operates in a non-homogenous situation, meaning that packets from the sources may have unequal chance of being received successfully, i.e., $C_H(S_i, D) \ne C_H(S_1, D)$ for some set of sources H and some i. As seen in Section 4, Theorem 1 implies that, given a general wireless network of K sources and a destination, our problem reduces to finding the optimal subset H^* of sources that, when they transmit simultaneously in a time slot, maximizes the network throughput

$$T = \sum_{S \in H^*} C_{H^*}(S, D)$$

For a given set of K sources, there are $2^K - 1$ non-empty subsets of sources. By comparing the throughput values among these $2^K - 1$ subsets, the optimal subset H^* of sources can be found. Thus, exhaustive search has $O(2^K)$ complexity.

In the following, we present 2 heuristic algorithms of greatly reduced complexity. Our numerical evaluation for numerous network instances of small and moderate size shows that they produce optimal or nearly optimal solutions. In both of these algorithms, candidate transmission sets are produced by choosing an initial transmitting node, and then adding additional nodes into the time slot until the resulting throughput does not increase further. The best such transmission set is the one that results in the largest value of throughput. The algorithms for constructing the candidate

transmission sets differ in terms of the order according to which the sources are added into the time slot.

Algorithm 1: Systematic Node Selection

- Label the K sources according to decreasing values of their throughput (under the assumption that each node transmits alone in a time slot), i.e., S_1 has the highest throughput and S_K the lowest. This can be done via sorting, which has $\mathrm{O}(K \log K)$ complexity.
- For each i, $1 \leq i \leq K$, form the following candidate transmission set of consecutive sources

$$A_i = \{S_i, S_{i+1}, \ldots, S_{i+n_i-1}, S_{i+n_i}\}$$

  where $i + n_i \leq K$. Note that there are no gaps in A_i, i.e., $|A_i| = n_i + 1$. The sources $S_i, S_{i+1}, \ldots, S_{i+n_i}$ are added one by one into the time slot, where S_{i+n_i} is the last source beyond which the throughput does not increase further. Let $T(A_i)$ be the throughput obtained when the sources in A_i transmit in the time slot.
- Finally, we choose the transmission set A^* that yields the maximum throughput, i.e., the chosen A^* satisfies $T(A^*) = \max\{T(A_i), 1 \leq i \leq K\}$. It can be shown that the overall complexity of Algorithm 1 is $\mathrm{O}(K^2)$.

Note that the candidate transmission sets for Algorithm 1 contain no gaps, when the sources are ordered from highest expected received power to lowest. Restricting the class of policies that must be examined in this manner reduces the complexity of the algorithm.

Algorithm 2: Greedy Node Selection

As with Algorithm 1, K candidate transmission sets are constructed, each using one of the K sources as a starting point. Each additional source is chosen with the greedy objective of maximizing the throughput in the slot. Unlike Algorithm 1, the numbering of the sources in this algorithm is arbitrary, i.e., unrelated to their throughput in isolation. The basic idea is, at each step, to add the source that yields the maximum throughput into the time slot. More specifically,

- For each i, $1 \leq i \leq K$, perform the following procedure:
  - Step 1: Add source S_i as the initial source into the time slot.
  - Step k: Add into the slot the source that yields the maximum throughput computed up to this step.
  - The algorithm stops when the throughput does not increase further.
- Repeat the above procedure with S_i as initial source, for each i, $1 \leq i \leq K$. Let $T(S_i)$ be the resulting throughput obtained when S_i is the initial source. We then choose the initial source that yields the maximum throughput, i.e., the chosen initial source S_{i^*} satisfies $T(S_{i^*}) = \max\{T(S_i), 1 \leq i \leq K\}$. It can be shown that the overall complexity of Algorithm 2 is $\mathrm{O}(K^3)$.

We now compare the throughput performance for OPT (i.e., exhaustive search), Algorithm 1, and Algorithm 2. We assume the following:

- The path-loss exponent is $a = 3$.
- The wireless channel is affected by Rayleigh fading with Rayleigh parameter $v(S, D) = 1$.
- The received power factor $g(S, D)$ is given by (3).
- The transmit power is $P_{\mathrm{tx}}(S) = 1$ for all source nodes S. The receiver noise power at destination D is $P_{\mathrm{noise}}(D) = 10^{-4}$.

We now study a stationary wireless network as shown in Fig. 3, which has a destination D and K sources (S_i). Assume that the sources are located randomly in the circle centered at (0, 0) and of radius $r = 5$. The destination is located at $(x_D, 0)$. In the following, we show the throughput T versus the SINR threshold β for various network sizes and topology configurations. The throughput values are averaged over 100 random network instances. Smaller values of β result in higher throughput T, as expected.

Consider a small network with $K = 10$ sources. First, let $x_D = 0$, i.e., the destination is located at the center of the circular region in which the sources are distributed. The performance results are shown in Fig. 4, which shows that our 2 heuristic algorithms (which have polynomial-time complexity) perform almost identically to OPT (which is an expensive exhautsive search). In fact, our numerical results show that Algorithm 1 performs identically to OPT, and Algorithm 2 also performs identically to OPT with the exception of a few network instances.

Next, let $x_D = 10$, i.e., the destination is outside the circle of radius $r = 5$. As shown in Fig. 5, the throughput results are lower than those for the case of $x_D = 0$. This is because the distances between the sources and the destination are larger (while the receiver noise power at destination D still maintains at $P_{\mathrm{noise}}(D) = 10^{-4}$), which imply that the SINR at the destination is now reduced. Again, our 2 heuristic algorithms perform almost identically to OPT. Similar observations (which are not shown here) also hold for networks of moderate size (e.g., $K = 20$ and $K = 30$).

We now consider a network with $K = 100$ nodes. It is not feasible to apply OPT, which has high computational complexity, to the network with this large size. The throughput results are shown in Fig. 6 (for $x_D = 0$) and Fig. 7 (for $x_D = 10$) for our polynomial-time Algorithms 1 and 2. The 2 algorithms perform almost identically, with the exception of a few network instances where Algorithm 1 slightly outperforms Algorithm 2.

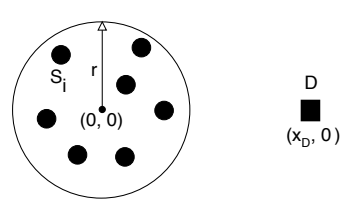

Fig. 3 A wireless network with K sources (S_i) and a destination (D).

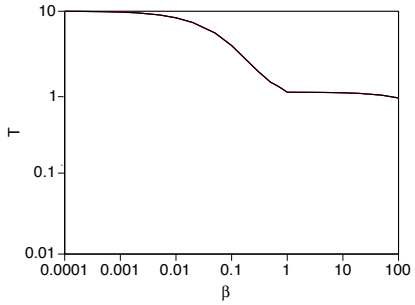

Fig. 4 Throughput (T) vs SINR threshold (β) for $K = 10$ and $x_D = 0$.

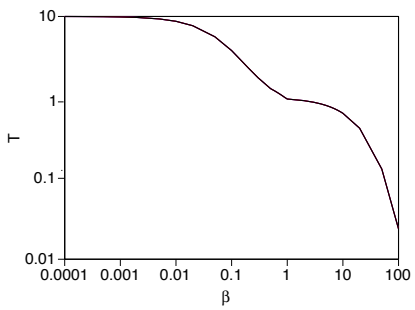

Fig. 5 Throughput (T) vs SINR threshold (β) for $K = 10$ and $x_D = 10$.

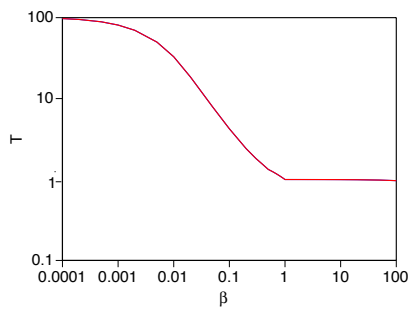

Fig. 6 Throughput (T) vs SINR threshold (β) for $K = 100$ and $x_D = 0$.

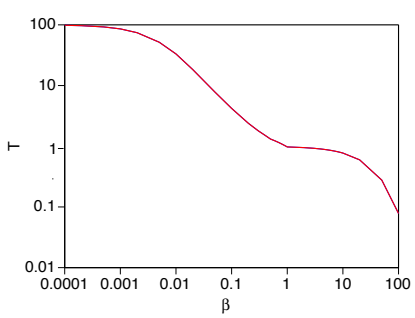

Fig. 7 Throughput (T) vs SINR threshold (β) for $K = 100$ and $x_D = 10$.

Remark 2 One of the algorithms (OPT, Algorithm 1, or Algorithm 2) in this section can be used to find a subset H of sources that transmit in a single time slot. For the case of $|H| < K$, we can use the algorithm repeatedly (with a modified set of available sources) to allow all the sources to transmit in multiple time slots. For example, suppose that the total number of sources is $K = 6$, and suppose that the subset of sources that transmit in each time slot is found to be $H = \{S_2, S_5\}$. We can then transmit S_2 and S_5 in time slot 1. We apply the same algorithm to the reduced source set $\{S_1, S_3, S_4, S_6\}$ to find the next subset of sources for transmission in time slot 2, etc. In this way, as a method for fairness, all the sources can transmit in multiple time slots. □

7 Summary

For the MAC problem considered in this paper, throughput in a single slot is maximized by the choice of an appropriate transmission set, rather than via a randomized approach in which the sources transmit with some probability. This reduces the optimization problem from a continuous one to a finite one. As shown in our studies, the proposed heuristic algorithms for finding the transmission sets not only have low complexity, but also perform almost identically to the optimal sets obtained by expensive exhaustive search. Further, our transmission method for a single time slot (for providing transmission opportunities to a subset of sources) can be extended to multiple time slots (for providing transmission opportunities to all sources, i.e., a method for fairness).

Acknowledgment

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